An Analytical Method for Ride Quality of Flexible Airplanes

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A new and easily used method of aircraft ride quality analysis is developed, and its use is illustrated by a numerical example using DC-8 airplane equations of motion. The method readily allows any type of stability augmentation system to be included for good handling qualities. A particularly powerful method of specifying and achieving all roots of a desired closed-loop characteristic equation by use of full state variable feedback is described and applied to the DC-8 example.

Introduction

THE advent of control-configured vehicles (CCV) and active control technology (ACT) in recent years has made it feasible to design advanced civil transport aircraft with acceptable performance, stability, handling qualities, and riding qualities throughout the mission profiles.[1-3] However, the need for much research remains before design criteria and Federal Aviation Regulations Part 25 certification standards can be set, bringing this technology to production application.

Relaxed static stability (RSS) is the CCV function most likely to result in the largest, most immediate payoff and first-reach production status for future large transports. RSS allows more aft c.g. location and reduced empennage size, with associated drag and weight reductions, by use of stability augmentation systems to restore acceptable handling qualities. These advanced airplanes must be optimized for good ride, as well as performance, and no design criteria exist to achieve this in other than a trial and error fashion.

An investigation is being conducted into the relationships and tradeoffs between handling qualities and ride quality for large, highly elastic airplanes with RSS implemented, but with no active elastic mode suppression control system in use. An important question to be answered is how sensitive the ride quality is (in turbulence) to the type of stability augmentation used, and to variations in handling qualities parameters such as short-period and dutch-roll damping ratios and undamped natural frequencies.

The commonly used ride quality parameter is the vertical and lateral normal acceleration rms responses at selected fuselage stations. In terms of perturbation of flight path angle γ; pitch angle θ; roll-angle ϕ; yaw angle ψ; sideslip angle β; symmetric mode shapes and generalized coordinates ϕ_i and ξ_i; and antisymmetric shapes and coordinates ϕ_j and ξ_j, the vertical and lateral normal acceleration load factors as a function of fuselage station ℓ_x (positive forward of the c.g.) are

$$n_z(\ell_x,t) = \frac{1}{g}\left[U_0\dot{\gamma} + \ell_x\ddot{\theta} - \sum_{i=1}^{n}\phi_i(\ell_x)\ddot{\xi}_i(t)\right] \tag{1}$$

$$n_y(\ell_x,t) = \frac{1}{g}\left[g\phi - (\dot{\beta}+\dot{\psi})U_0 - \ell_x\ddot{\psi} - \sum_{j=1}^{m}\phi_j(\ell_x)\ddot{\xi}_j(t)\right] \tag{2}$$

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The usual approach for studying ride quality analytically has been to use the frequency domain techniques, which consist of calculating power spectral density functions and integrating them to arrive at mean-square response values. Aircraft transfer functions and turbulence power spectral density models are part of such analyses.[4,5] A different approach is taken in the present study by formulating the problem entirely in the time domain, using state vector and matrix operations, which results in a fast, compact, and effective alternate method for ride quality studies of flexible airplanes.

State Vector Equations

An easily formulated and convenient form of aircraft dynamic equations of motion is the vector-matrix form

$$\dot{x} = Ax + Bu + G\eta_g \tag{3}$$

where x is an $(n\times 1)$ matrix of the physical aircraft variables, u is an $(m\times 1)$ control input vector, and η_g is a $(k\times 1)$ matrix of the gust velocity state variables. A, B, and G are $(n\times n)$, $(n\times m)$, and $(n\times k)$ coefficient matrices, respectively.

The variables in the gust matrix η_g satisfy the vector-matrix equation (4), where coefficient matrices A_g and G_g can be determined from gust velocity power spectral density math models; η is scalar, zero mean, unit white noise

$$\dot{\eta}_g = A_g\eta_g + G_g\eta \tag{4}$$

By augmenting the aircraft state vector x with the gust states, an alternate form of Eq. (3) is

$$\begin{bmatrix}\dot{x}\\ \dot{\eta}_g\end{bmatrix} = \begin{bmatrix}A & G\\ 0 & A_g\end{bmatrix}\begin{bmatrix}x\\ \eta_g\end{bmatrix} + \begin{bmatrix}B\\ 0\end{bmatrix}u + \begin{bmatrix}0\\ G_g\end{bmatrix}\eta \tag{5}$$

For almost all stability augmentation system (SAS) control laws, from simple rate feedback to full state, the control input vector u can be expressed as

$$u = -Kx \tag{6}$$

where K is an $(m\times n)$ matrix of feedback gains. By use of Eq. (6), Eqs. (3) and (5) become Eqs. (7) and (8):

$$\dot{x} = [A - BK]x + G\eta_g \tag{7}$$

$$\begin{bmatrix}\dot{x}\\ \dot{\eta}_g\end{bmatrix} = \begin{bmatrix}A-BK & G\\ 0 & A_g\end{bmatrix}\begin{bmatrix}x\\ \eta_g\end{bmatrix} + \begin{bmatrix}0\\ G_g\end{bmatrix}\eta \tag{8}$$

To simplify the notation in what follows, let x^*, D, and F now be used to indicate the augmented system of Eq. (8), or

$$\dot{x}^* = Dx^* + F\eta \tag{9}$$

Ride Quality Analysis

The vertical or lateral load factor [i.e., Eqs. (1) or (2)] can be expressed as

$$n_{z,y}(\ell_x,t) = Px^* \tag{10}$$

where P is a $(1\times n+k)$ row matrix of deterministic coefficients (different for n_z and n_y), which makes n_z or n_y a scalar. The mean square or expected value of $n^2_{z,y}$ is obtained by squaring and averaging.

$$n^2_{z,y}(\ell_x,t) = [Px^*][Px^*] = [Px^*][Px^*]' \tag{11}$$

where [ ]' indicates the matrix transpose. Since $[Px^*]$ is a scalar, then $[Px^*]' = [Px^*]$, and

$$\begin{aligned} n^2_{z,y}(\ell_x,t) &= [Px^*][x^{*\prime}P'] = Px^*x^{*\prime}P' \\ &= [p_1,p_2,\ldots,p_{n+k}][x^*x^{*\prime}]\begin{bmatrix} p_1 \\ p_2 \\ \cdot \\ \cdot \\ p_{n+k}\end{bmatrix} \end{aligned} \tag{12}$$

The mean square value then is the expected value $E[\,]$:

$$E[n^2_{z,y}(\ell_x,t)] = PE[x^*x^{*\prime}]P' \tag{13}$$

Where $E[x^*x^{*\prime}]$ is a symmetric square $(n+k)\times(n+k)$ state covariance matrix, which can be determined as follows. From Eq. (9),

$$E[\dot{x}^*x^{*\prime}] = DE[x^*x^{*\prime}] + FE[\eta x^{*\prime}] \tag{14}$$

Also,

$$\dot{x}^{*\prime} = [Dx^*]' + [F\eta]' = x^{*\prime}D' + \eta'F' \tag{15}$$

and

$$x^*\dot{x}^{*\prime} = x^*x^{*\prime}D' + x^*\eta'F' \tag{16}$$

$$E[x^*\dot{x}^{*\prime}] = E[x^*x^{*\prime}]D' + E[x^*\eta']F' \tag{17}$$

It can be shown with fundamental stochastic analysis that, for a linear system driven by unit white noise, as Eq. (9), the correlation between x^* and η is[6]

$$E[\eta x^{*\prime}] = F'/2 \tag{18}$$

and

$$E[x^*\eta'] = F/2 \tag{19}$$

Adding Eqs. (14) and (17), and using (18) and (19), gives

$$E[\dot{x}^*x^{*\prime}] + E[x^*\dot{x}^{*\prime}] = DE[x^*x^{*\prime}] + E[x^*x^{*\prime}]D' + FF' \tag{20}$$

For statistically time stationary (constant rms value) systems,

$$(d/dt)E[x^*x^{*\prime}] = 0 = E[\dot{x}^*x^{*\prime}] + E[x^*\dot{x}^{*\prime}] \tag{21}$$

This is true in the present case as $t\to\infty$, if the system in Eq. (9) is stable. Thus, Eq. (20) becomes the algebraic matrix Riccati equation (22).

$$DE[x^*x^{*\prime}] + E[x^*x^{*\prime}]D' + FF' = 0 \tag{22}$$

Given D and F, Eq. (22) has a unique solution for the elements of the symmetric covariance matrix, which then can be used in the evaluation of Eq. (13). Equation (22) usually must be solved numerically, and various computer algorithms exist for doing so. One method, with fast convergence on aircraft ride quality problems, is described in Ref. 7.

Scalar Control SAS

Most conventional aircraft stability augmentation control philosophies use a single control input, usually elevator in the longitudinal case and rudder in the lateral-directional case. Since the ride quality analysis can be divided into separate longitudinal and lateral-directional cases, then the feedback gain matrix K in Eq. (6) becomes a $(1\times n)$ row matrix.

In the scalar control input case, a powerful method for setting the feedback gains to achieve specified handling qualities is first to transform the aircraft states to phase variable canonical (companion) form.[8] If the system in Eq. (3) is completely state controllable, then there exists a transformation matrix T in $x=Ty$, which yields a phase variable canonical form in the vector y.

$$T\dot{y} = ATy + Bu + G\eta_g \tag{23}$$

and

$$\dot{y} = [T^{-1}AT]y + [T^{-1}B]u + [T^{-1}G]\eta_g \tag{24}$$

where

$$[T^{-1}AT] = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 \\ \cdot & & & & \ddots & \cdot \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ \cdot & & & & & 0 \\ 0 & & & & & 1 \\ -d_0 & -d_1 & \cdots & & & -d_{n-1} \end{bmatrix} \tag{25}$$

and

$$[T^{-1}B] = \begin{bmatrix} 0 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 1 \end{bmatrix} \tag{26}$$

The d coefficients in Eq. (25) are the coefficients of the bare-airframe characteristic equation (27).

$$|sI-A| = s^n + d_{n-1}s^{n-1} + \ldots + d_1 s + d_0 = 0 \tag{27}$$

The $(n\times n)$ matrix T is formed as follows[8]

$$T[t_1 t_2 \cdots t_n] \tag{28}$$

where

$$\begin{aligned} t_n &= B \\ t_{n-1} &= At_n + d_{n-1}t_n \\ t_{n-2} &= At_{n-1} + d_{n-2}t_n \\ &\vdots \\ t_1 &= At_2 + d_1 t_n \end{aligned} \tag{29}$$

The scalar control, or SAS control law, is

$$u=-Kx=-KTy \tag{30}$$

and Eq. (24) becomes

$$\dot{y}=T^{-1}[A-BK]Ty+[T^{-1}G]\eta_g=A^*y+[T^{-1}G]\eta_g \tag{31}$$

A^* is given by Eq. (32).

$$A^*=\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \ddots & \cdot \\ \cdot & \cdot & & \ddots & \cdot \\ \cdot & \cdot & & & \cdot \\ \cdot & \cdot & & & 0 \\ 0 & 0 & \cdots & & 1 \\ (-d_0+k_1) & (-d_1+k_2) & \cdots & & (-d_{n-1}+k_n) \end{bmatrix} \tag{32}$$

The last row of A^*contains the coefficients of the desired closed-loop characteristic equation. Handling qualities are strongly influenced by the roots of the characteristic equation. Therefore, if the desired closed-loop characteristic equation is specified by selecting all of the roots for good handling qualities, the coefficients e, as in Eq. (33), then are known and are related to those in A^* by Eq. (34).

$$s^n+e_{n-1}s^{n-1}+\ldots+e_1s+e_0=0 \tag{33}$$

$$\begin{aligned} -e_0 &= -d_0 + k_1 \\ -e_1 &= -d_1 + k_2 \\ &\vdots \\ -e_{n-1} &= -d_{n-1} + k_n \end{aligned} \tag{34}$$

The k's then are calculated easily, given the d's and e's. The k row matrix is obtained from Eq. (35), using Eq. (26).

$$-T^{-1}BKT=\begin{bmatrix} 0 & 0 & \ldots & 0 \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ 0 & 0 & \ldots & 0 \\ k_1 & k_2 & \ldots & k_n \end{bmatrix} \tag{35}$$

$$K=[K_1,K_2,\ldots K_n]=[-k_1,-k_2,\ldots-k_n][T^{-1}] \tag{36}$$

Calculating the BK square matrix allows determination of the augmented D matrix in Eqs. (8) and (9). The algebraic matrix Riccati equation (22) can be solved numerically for the covariance matrix $E[x^*x^{*\prime}]$, and the mean square load factor can be obtained from Eq. (13).

Although full-state feedback usually is not a practical SAS philosophy for implementation, particularly for high-order systems, it does provide a means for parametric studies of closed-loop damping ratio and frequency effects on ride quality, which is a major objective of our on-going research. It should be stressed that the simpler more practical SAS philosophies, such as a single-rate feedback loop, are included easily in the ride quality method.

Numerical Example

As a simple example of the ride quality analysis, consider the two-degree-of-freedom short-period approximate equations of motion for the DC-8 jet transport, cruising at Mach 0.84 at 33,000 ft. with SAS implemented by elevator control δ_e. The elastic equations have been omitted to keep the example brief.

The state variables are $x_1=w$, plunge velocity; and $x_2=\dot{\theta}$, pitch rate. Let the full-state variable feedback SAS be

$$\delta_e=-Kx=-k_1x_1-K_2x_2 \tag{37}$$

Equations (3) and (4) are

$$\dot{x}=\begin{bmatrix} -0.806 & 824.2 \\ -0.0107 & -1.344 \end{bmatrix}x+\begin{bmatrix} -34.6 \\ -4.572 \end{bmatrix}\delta_e + \begin{bmatrix} -0.806 & 0 & 0 \\ -0.0107 & 0 & -0.924 \end{bmatrix}\eta_g \tag{38}$$

$$\dot{\eta}_g=\begin{bmatrix} -0.471 & -0.237 & 0 \\ 0 & -0.471 & 0 \\ 0 & -0.00131 & -5.02 \end{bmatrix}\eta_g+\begin{bmatrix} 1.1887 \\ 1 \\ 0.00656 \end{bmatrix}\eta \tag{39}$$

Vertical and pitching Dryden gust spectra of 1-fps intensity are used to arrive at Eq. (39). The bare-airframe characteristic equation (27) is

$$|sI-A|=s^2+2.15s+9.893=0 \tag{40}$$

which gives a short-period frequency and damping ratio of $\omega_{sp}=3.145$ rad/sec and $\zeta_{sp}=0.342$.

A SAS will be used to augment these terms to $\omega_{sp}=4.5$ rad/sec and $\zeta_{sp}=0.6$, which requires that the e coefficients of Eq. (34) be $e_0=20.25$ and $e_1=5.4$. From Eq. (40), $d_0=9.893$ and $d_1=2.15$. The k's of Eqs. (34) and (36) are $-k_1=10.357$ and $-k_2=3.25$. From Eq. (29), the T matrix is

$$T=\begin{bmatrix} -3870.52 & -34.6 \\ -3.319 & -4.572 \end{bmatrix} \tag{41}$$

$$T^{-1}=\begin{bmatrix} -0.00026 & 0.00197 \\ 0.00019 & -0.22015 \end{bmatrix} \tag{42}$$

From Eq. (36), $K_1=-0.00208$ and $K_2=-0.695$. BK is

$$BK=\begin{bmatrix} 0.071968 & 24.047 \\ 0.00951 & 3.17754 \end{bmatrix} \tag{43}$$

The D matrix is

$$D=\left[\begin{array}{cc|ccc} -0.878 & 800.153 & -0.806 & 0 & 0 \\ -0.0202 & -4.522 & -0.0107 & 0 & -0.924 \\ \hline 0 & 0 & -0.471 & -0.237 & 0 \\ 0 & 0 & 0 & -0.471 & 0 \\ 0 & 0 & 0 & -0.00131 & -5.02 \end{array}\right] \tag{44}$$

Computer solution of the Riccati Equation (22) yields the (5 × 5) covariance matrix $E[x^*x^{*\prime}]$:

$$E[x^*x^{*\prime}] = \left[\begin{array}{cc|ccc} 0.3928 & -0.000134 & -0.5584 & -0.5805 & -0.000201 \\ -0.000134 & 0.0000011 & -0.000102 & 0.0000246 & -0.0000013 \\ \hline -0.5584 & -0.000102 & 0.998 & 0.995 & 0.00114 \\ -0.5805 & 0.0000246 & 0.995 & 1.04 & 0.000948 \\ -0.000201 & -0.0000013 & 0.00114 & 0.000948 & 0.00000404 \end{array}\right] \tag{45}$$

The P row matrix in Eqs. (10) and (13) for the K_1 and K_2 SAS gains is

$$P = [(0.0273\text{-}0.00063\ell_x), (0.7478 - 0.1406\ell_x), (0.0251\text{-}0.00033\ell_x), 0, (-0.0287\ell_x)] \tag{46}$$

The vertical rms load factor given by the square root of Eq. (13) is, therefore, a function of fuselage station through the length parameter ℓ_x in Eq. (46). The DC-8 used in this example is 150 ft long. The load factors for values of ℓ_x equal to 55, 0, and −90 ft, corresponding to nose, center of gravity, and tail fuselage stations, are given in Table 1.

The units on $(n_z)_{\rm rms}$ are g's/fps gust velocity intensity. For moderate turbulence intensity of about 6 fps, the load factors at nose, c.g., and tail become 0.0678, 0.0726, and 0.144 g's, respectively. These levels certainly correspond to uncomfortable passenger ride if they persist for more than a few minutes because of clear air turbulence at the 33,000 ft alt.

The airplane orthogonal elastic vibration modes are incorporated readily into this method of ride quality analysis, and their contribution to ride assessed. All that is needed are the additional elastic equations of motion in the plant dynamics of Eq. (3). We have done this for the B-52H and B-1 aircraft (results appear elsewhere[9]).

Table 1 DC-8 vertical load factors

ℓ_x	Fuselage position	$E[n_z^2(\ell_x,t)]$	$(n_z)_{\rm rms}$
55	nose	1.285×10^{-4}	0.0113
0	center of gravity	1.473×10^{-4}	0.0121
−90	tail	5.739×10^{-4}	0.0240

Conclusions

A new and easily used method of aircraft ride quality analysis has been formulated, and its use is illustrated by a numerical example on the DC-8 airplane. The technique readily allows any type of stability augmentation system to be included for good handling qualities. A particularly powerful method for specifying and achieving all of the roots of a desired closed-loop characteristic equation by use of full-state-variable feedback is developed and illustrated by application to the DC-8 example. The effect on ride of different SAS philosophies and various damping ratios and frequencies can be studied easily with this formulation.

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